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SUPERSONIC UNDEREXPANDED JET IMPINGEMENT UPON FLAT PLATE

I. A. BELOV, I. P. GINZBURG and L. I. SHUB
Mechanical Institute, Leningrad, USSR

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Abstract—Approximate solution of the problem on an axisymmetrical underexpanded supersonic jet impinging upon a normal flat plate within the initial length of the jet is presented. The results of the calculation obtained in the ideal liquid treatment show the existence of appreciable vorticity in a subsonic region close to the plate. The received values of the radial velocity gradient at the stagnation point and the vorticity distribution are then used for calculation of a viscous flow and heat transfer in the vicinity of the stagnation point. It is shown that the value of the heat flux at the stagnation point found with account for jet vorticity is as large as 3–5 times that of a uniform heat flux that is verified experimentally.

NOMENCLATURE

r, z, cylindrical coordinates;
v_r, v_z, velocity components;
T, temperature;
p, pressure;
ρ, density;
T_0, p_0, ρ_0, stagnation temperature, pressure, density, respectively;
v_{max}, maximum velocity;
L, nozzle-to-plate spacing;
n, pressure ratio;
M, Mach number;
r_{cr}, h, coordinates of a sonic point;
Q, mass flow;
A_n, coefficients in the jet vorticity spectrum (equation (28));
k_n, wave number corresponding to the n-th wave length of the vorticity spectrum;
q, heat flux.

Greek symbols

ε, distance from the plate to a gas dynamic line;
$\varkappa$, specific heat ratio;
β, constant defined by equation (24);
φ, entropy function (equation (4));
ψ, stream function;
Ω, vorticity;
ν, kinematic viscosity;
λ_n, wave length;

$$\alpha(M) = \left(1 + \frac{\varkappa - 1}{2} M^2\right)^{-\frac{1}{2}};$$

$$\tau(M) = \left(1 + \frac{\varkappa - 1}{2} M^2\right)^{-1};$$

$$\pi(M) = \left(1 + \frac{\varkappa - 1}{2} M^2\right)^{-\varkappa/(\varkappa - 1)}.$$

Subscripts

0, along the plate surface for an ideal liquid;
w, on the wall;
1, on gas dynamic lines (shock wave; slip line);
c, refers to the triple point;
∞, refers to the outer edge of the shear layer of the plate.

INTRODUCTION

THE PROBLEM of heat transfer between a supersonic jet and a flat plate normal to the jet axis is one of the most complicated problems of gas dynamics to be solved at present. Despite numerous experimental data which are often contradictory theoretical information now available on the above problem is scanty. Empirical dimensionless relations for the Nusselt number of the type $Nu = Nu\,(Pr, Re)$ were not and will not be able to give an idea of the jet-plate heat exchange since such important factors as nonuniformity of the flow, jet turbulence, etc., have not been taken into account in the previous investigations. This may be a quite possible explanation for the existing contradiction in the experimental results on heat flux to the plate reported by different authors as well as for a great discrepancy (up to 10 times) between experimental data and theoretical predictions. The experimental works recently reported on heat transfer of a jet, although showing some progress, are based on the same dimensionless relations for a uniform flow near the stagnation point with a correction for turbulence and so reduce a jet impingement problem to a model problem on uniform flow with a known turbulence level [1].

For an essential nonuniformity of the flow peculiar to a supersonic jet this approach may lead to considerable errors in heat flux calculations.

Estimation of the flow nonuniformity effect on heat transfer of an axisymmetrical supersonic underexpanded jet with the normal flat plate is the purpose of the present work. The flow field in the jet impingement region is found from the solution of the ideal problem by the integral relationship procedure. The viscous flow near the plate as well as heat transfer are analysed from the solution of the Navier–Stokes equations by the Fourier method. A steady flow is assumed, the pressure ratio is varying in the range $2 \leqslant n \leqslant 30$, the analysis is restricted by the consideration of the jet impingement within the first "cell" of the jet.

CALCULATION OF THE SHOCK WAVE CONFIGURATIONS AND A FLOW IN THE IMPINGEMENT REGION

The calculation includes the impingement region bounded by the detached CE and the reflected CD shocks, the jet boundary DM, the axis of symmetry OE, the part of the plate OP and the limiting characteristics TP and TM

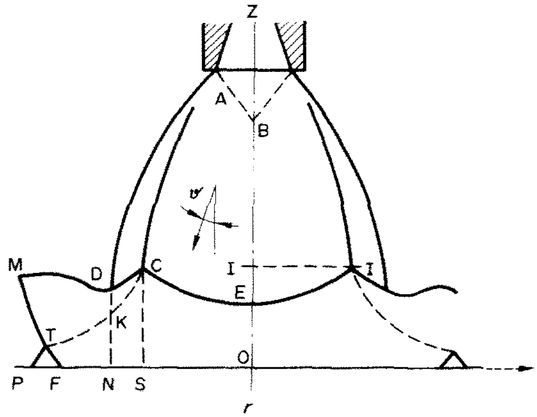

FIG. 1. Flow pattern.

(Fig. 1). In the cylindrical coordinates the governing system of equations is

$$\frac{\partial}{\partial r}(\rho r v_r) + \frac{\partial}{\partial z}(\rho r v_z) = 0, \tag{1}$$

$$\frac{\partial}{\partial r}(\rho r v_r v_z) + \frac{\partial}{\partial z}\left[r\left(\rho v_z^2 + \frac{\varkappa - 1}{2\varkappa}p\right)\right] = 0, \tag{2}$$

$$v_r^2 + v_z^2 + \frac{p}{\rho} = 1 \tag{3}$$

$$\frac{p}{\rho^{\varkappa}} = \varphi[\psi(r, z)], \tag{4}$$

where ρ and p are based on the corresponding stagnation values ρ_0 and p_0; v_z, v_r on the maximum velocity $v_{\max}$, the linear dimensions on the radius of the outlet section of the nozzle.

The equation of the shock generatrix

$$\frac{d\varepsilon}{dr} = -\operatorname{ctg}\sigma, \qquad 0 \leqslant r \leqslant r_C, \tag{5}$$

and the equation of the slip line

$$\frac{d\varepsilon}{dr} = -\operatorname{ctg}\theta, \qquad r_C < r < r_B \tag{6}$$

are to be added to equations (1)–(4) where ε is the distance from the shock (slip line); σ and θ

are the slopes to the shock and a velocity vector along CT to the z-axis, respectively.

To avoid simultaneous calculation of subsonic and supersonic flows, the slip line is approximated by the following polynomial

$$\varepsilon = a_1 r^2 + a_2 r + a_3, \tag{7}$$

the coefficients of which are to be found later.

The boundary conditions for the initial system of equations are imposed at the plate surface and the symmetry axis

$$\varphi = \varphi_0, \quad v_z = 0, \quad z = 0, \tag{8}$$

$$\varphi = \varphi_0, \quad \sigma = \frac{\pi}{2}, \quad \varepsilon = \varepsilon_0, \quad v_r = 0, \quad r = 0, \tag{9}$$

and at the shocks, the locations of which are unknown

$$\frac{v_{1z}}{V} = -\cos(\sigma - \theta)\cos\sigma - \sin(\sigma - \vartheta) \times \sin\sigma \frac{(\varkappa - 1)M^2 \sin^2(\sigma - \vartheta) + 2}{(\varkappa + 1)M^2 \sin^2(\sigma - v)},$$

$$\frac{v_{1r}}{V} = \sin\sigma\cos(\sigma - \vartheta)\cos\sigma \times \frac{(\varkappa - 1)M^2 \sin^2(\sigma - \vartheta) + 2}{(\varkappa + 1)M^2 \sin^2(\sigma - \vartheta)},$$

$$p_1 = p\left[\frac{2\varkappa}{\varkappa + 1} M^2 \sin^2(\sigma - \vartheta) - \frac{\varkappa - 1}{\varkappa + 1}\right],$$

$$\rho_1 = \rho \frac{(\varkappa + 1)M^2 \sin^2(\sigma - \vartheta)}{(\varkappa - 1)M^2 \sin^2(\sigma - \vartheta) + 2},$$

$$\varphi = \left[\frac{2\varkappa}{\varkappa + 1} M^2 \sin^2(\sigma - \vartheta) - \frac{\varkappa - 1}{\varkappa + 1}\right] \times \left[\frac{\varkappa - 1}{\varkappa + 1} + \frac{2}{(\varkappa + 1)M^2 \sin^2(\sigma - \vartheta)}\right]^{\varkappa} \tag{10}$$

where

$$V = \sqrt{(v_r^2 + v_z^2)} = \sqrt{\frac{\frac{\varkappa - 1}{2} M^2}{1 + \frac{\varkappa - 1}{2} M^2}}.$$

The boundary conditions for a subsonic flow flowing out of the impingement region are not put here; they are substituted by the conditions of the regularity solution at the singular points which result from the transition of equations (1)–(4) from the elliptical form to the hyperbolic one.

The method of integral relationships is used to solve equations (1)–(4). Consider the first approximation of this method. Integration of equations (1) and (2) with respect to z from 0 to ε and approximation of the integrand function by the linear relations yield the following integral relations

$$\frac{dB_1}{dr} + \left(\frac{1}{r} - \frac{1}{\varepsilon}\frac{d\varepsilon}{dr}\right) B_1 + \frac{2}{\varepsilon}(A_1 - A_0) = 0,$$

$$\frac{dC_0}{dr} + \frac{dC_1}{dr} + (C_0 - C_1)\frac{1}{\varepsilon}\frac{d\varepsilon}{dr} + \frac{C_0 + C_1}{r} + \frac{2}{\varepsilon} D_1 = 0, \tag{11}$$

where

$$A = \rho v_z^2 + \frac{\varkappa - 1}{\varkappa + 1} p; \qquad B = \rho v_r v_z;$$

$$C = \rho v_r; \qquad D = \rho v_z,$$

the subscript 0 refers to the flow parameters at the plate, the subscript 1 refers to parameters on the gas dynamic lines.

It is assumed that the incoming flow up to the shock EC may be approximated by the flow from a three-dimensional source [2]

$$\eta = \sqrt{[r^2 + (c - \varepsilon)^2]} = \sqrt{\varphi}\left[\sqrt{\left(\frac{q(M_B)}{q(M)}\right)} - 1\right];$$

$$\vartheta = \operatorname{arctg}\frac{r}{L - \varepsilon}, \tag{12}$$

where M_B is the Mach number at the point B where the axis of symmetry crosses the first characteristic of the second family from the edge of the nozzle (Fig. 1); c is the distance from the point B to the plate; L is the nozzle-to-plate spacing

$$q(M) = M\left[\frac{\varkappa+1}{2}\Big/1+\frac{\varkappa-1}{2}M^2\right]^{(\varkappa+1)/2(\varkappa-1)}$$

The location of the free jet shock wave EC is given by the following equation [3]

$$r = 1 + (L - \varepsilon_C)\,\mathrm{tg}\,(\vartheta_H - \alpha_H) - \left[1 - r_{\mathrm{M.D.}} + x_{\mathrm{M.D.}}\,\mathrm{tg}\,(\vartheta_H - \alpha_H)\right]\left(\frac{L-\varepsilon_C}{x_{\mathrm{M.D.}}}\right)^2, \tag{13}$$

where ε_C is the distance from the point C to the plate; $r_{\mathrm{M.D.}}$ is the radius of the Mach disc in the free jet; $x_{\mathrm{M.D.}}$ is the distance from the Mach disc to the outlet section nozzle.

The Mach number M and the slope angle of the velocity vector to the z-axis is chosen to be the dependent variables (ϑ up and θ down the shock). Then calculating equations (12) and (13) as well as the free jet flow parameters by the method described in [3], we can write relations (11) for the region up to the triple point C as

$$P_1\frac{\mathrm{d}M}{\mathrm{d}r} + P_2\frac{\mathrm{d}\sigma}{\mathrm{d}r} + P_3\frac{\mathrm{d}\vartheta}{\mathrm{d}r} + P_4 = 0, \tag{14}$$

$$\frac{\mathrm{d}M_0}{\mathrm{d}r} + Q_1\frac{\mathrm{d}M}{\mathrm{d}r} + Q_2\frac{\mathrm{d}\sigma}{\mathrm{d}r} + Q_3\frac{\mathrm{d}\vartheta}{\mathrm{d}r} + Q_4 = 0, \tag{15}$$

where

$$P_1 = \frac{1}{2}\frac{\tau(M)}{M}(2 - M^2)\left[\frac{1}{f}\sin 2\sigma\cos^2(\sigma-\vartheta) - \sin 2(\sigma-\vartheta)\cos 2\sigma - f\sin 2\sigma\sin^2(\sigma-\vartheta)\right] + \frac{\sin 2\sigma}{(\varkappa+1)M^2\sin^2(\sigma-\vartheta)}\times\left[\frac{1}{f^2}\cos^2(\sigma-\vartheta) + \sin^2(\sigma-\vartheta)\right],$$

$$P_2 = \tfrac{1}{2}\cos(\sigma-\vartheta)\cos(3\sigma-\vartheta) - \cos(4\sigma - 2\vartheta) - f\sin(\sigma-\vartheta)\sin(3\sigma-\vartheta) + \frac{\sin 2\sigma\,\mathrm{ctg}\,(\sigma-\vartheta)}{(\varkappa+1)M^2\sin^2(\sigma-\vartheta)}\left[\frac{1}{f^2}\cos^2(\sigma-\vartheta) + \sin^2(\sigma-\vartheta)\right],$$

$$P_3 = \frac{1}{2f}\sin 2\sigma\sin 2(\sigma-\vartheta) + \cos 2(\sigma-\vartheta \times\cos 2\sigma + \frac{f}{2}\sin 2\sigma\sin 2(\sigma-\vartheta) - \frac{\sin 2\sigma\,\mathrm{ctg}\,(\sigma-\vartheta)}{(\varkappa+1)M^2\sin^2(\sigma-\vartheta)}\left[\frac{1}{f^2}\cos^2(\sigma-\vartheta) + \sin^2(\sigma-\vartheta)\right],$$

$$P_4 = \frac{1}{2}\left[\frac{1}{f}\sin 2\sigma\cos^2(\sigma-\vartheta) - \sin 2(\sigma-\vartheta)\times\cos 2\sigma - \sin 2\sigma\sin^2(\sigma-\vartheta)f\right]\times\left(\frac{1}{r} + \frac{\mathrm{ctg}\,\sigma}{\varepsilon}\right) - \frac{2}{\varepsilon}\left[\frac{1}{f}\cos^2(\sigma-\vartheta)\cos^2\sigma + \tfrac{1}{2}\sin 2(\sigma-\vartheta)\sin 2\sigma + f\sin^2(\sigma-\vartheta)\sin^2\sigma\right] - \frac{2}{\varepsilon}\frac{1}{\varkappa M^2}\left[\frac{2\varkappa}{\varkappa+1}M^2\sin^2(\sigma-\vartheta) - \frac{\varkappa-1}{\varkappa+1} - \frac{\pi(M_0)}{\pi(M)}\varphi_0^{-[1/(\varkappa-1)]}\right],$$

$$Q_1 = \frac{\pi(M)}{\alpha(M_0)\pi(M_0)\alpha(M)(1-M_0^2)}\varphi_0^{1/(\varkappa-1)}\left\{\tau(M)(1-M^2)\times\left[\frac{1}{f}\sin\sigma\cos(\sigma-\vartheta) - \sin(\sigma-\vartheta)\cos\sigma\right] + \frac{1}{f^2}\frac{2\sin\sigma\cos(\sigma-\vartheta)}{(\varkappa+1)M^2\sin^2(\sigma-\vartheta)}\right\},$$

$$Q_2 = M\frac{\pi(M)}{\pi(M_0)}\varphi_0^{1/(\varkappa-1)}\frac{1}{\alpha(M)\alpha(M_0)(1-M_0^2)}\times\left[\left(\frac{1}{f}-1\right)\cos(2\sigma-\vartheta)\right] + \frac{1}{f^2}\times\frac{2\sin\sigma\cos^2(\sigma-\vartheta)}{(\varkappa+1)M^2\sin^3(\sigma-\vartheta)},$$

$$Q_3 = -M\frac{\pi(M)}{\pi(M_0)}\varphi_0^{1/(\varkappa-1)}\frac{1}{\alpha(M)\alpha(M_0)(1-M_0^2)}$$

$$\times\left[\frac{1}{f^2}\frac{2\sin\sigma\cos^2(\sigma-\vartheta)}{(\varkappa+1)M^2\sin^2(\sigma-\vartheta)}-\frac{1}{f}\sin\sigma\right.$$
$$\left.\times\sin(\sigma-\vartheta)\cos(\sigma-\vartheta)\cos\sigma\right],$$

$$Q_4=\left\{\left(\frac{1}{r}-\frac{1}{\varepsilon}\operatorname{ctg}\sigma\right)\frac{M_0}{\tau(M_0)}\right.$$
$$+\frac{M\pi(M)\varphi_0^{1/(\varkappa-1)}}{\alpha(M)\pi(M_0)\alpha(M_0)}\left(\frac{1}{r}+\frac{1}{\varepsilon}\operatorname{ctg}\sigma\right)$$
$$\times\left[\frac{1}{f}\sin\sigma\cos(\sigma-\vartheta)-\sin(\sigma-\vartheta)\cos\sigma\right]$$
$$\left.-\frac{2}{\varepsilon}\left[\frac{1}{f}\cos(\sigma-\vartheta)\cos\sigma+\sin(\sigma-\vartheta)\sin\sigma\right]\right\}$$
$$\times(1-M_0^2)^{-1},$$

$$f=\frac{\varkappa-1}{\varkappa+1}+\frac{2}{(\varkappa+1)M^2\sin^2(\sigma-\vartheta)},$$

φ_0 is the value of the entropy function along the zeroth streamline.

Differential equations (14), (15) and (5) are used to determine the three unknowns: $M_0(r)$, $\sigma(r)$, $\varepsilon(r)$ within the zone $r\in\mathrm{OS}$ (Fig. 1). The initial conditions for these equations are

$$M_0(0)=0,\qquad \sigma_0(0)=\pi/2,\qquad \varepsilon(0)=\varepsilon_0.$$

The values of M and ϑ as well as their derivatives are obtained in accordance with the known flow field before the central shock (12); integration of equations (14), (15) and (5) is carried out up to the meeting the detached shock (equation (5)) with the free jet shock (equation (13)). Then the calculation of the flow parameters in the vicinity of the triple point C is done using the conditions of the pressure equality for the flow along the two sides of the slip line [4].

From the triple point and further to the periphery of the impingement region equations (11) are rewritten as

$$R_1\frac{dM_1}{dr}+R_2\frac{d\theta}{dr}+R_3=0, \qquad (16)$$

$$S_0\frac{dM_0}{dr}+S_1\frac{dM_1}{dr}+S_2\frac{d\theta}{dr}+S_3=0, \qquad (17)$$

where

$$R_1=\tfrac{1}{2}M_1\pi(M_1)(2-M_1^2)\tau(M_1)\varphi_1^{-1/(\varkappa-1)}\sin 2\theta,$$
$$R_2=M_1^2\pi(M_1)\cos 2\theta\varphi_1^{-1/(\varkappa-1)}$$
$$R_3=\frac{1}{2}\left(\frac{1}{r}+\frac{\operatorname{ctg}\theta}{\varepsilon}\right)M_1^2\pi(M_1)\sin 2\theta\varphi_1^{-1/(\varkappa-1)}$$
$$-\frac{2}{\varepsilon}\left[\frac{\pi(M_1)}{\varkappa}(1+\varkappa M_1^2\cos^2\theta)\varphi_1^{-1/(\varkappa-1)}\right.$$
$$\left.-\frac{\pi(M_0)}{\varkappa}\varphi_0^{-1/(\varkappa-1)}\right],$$
$$S_0=\pi(M_0)\alpha(M_0)(1-M_0^2)\varphi_0^{-1/(\varkappa-1)}$$
$$S_1=\pi(M_1)\alpha(M_1)(1-M_1^2)\varphi_1^{-1/(\varkappa-1)}\sin\theta,$$
$$S_2=M_1\frac{\pi(M_1)}{\alpha(M_1)}\cos\theta\varphi_1^{-1/(\varkappa-1)},$$
$$S_3=M_0\frac{\pi(M_0)}{\alpha(M_0)}\varphi_0^{-1/(\varkappa-1)}\left(\frac{1}{r}-\frac{\operatorname{ctg}\theta}{\varepsilon}\right)$$
$$+M_1\frac{\pi(M_1)}{\alpha(M_1)}\varphi^{-1/(\varkappa-1)}\left(\frac{1}{r}-\frac{\operatorname{ctg}\theta}{\varepsilon}\right)\sin\theta,$$

θ is the slope angle of the velocity vector and φ_1 is the entropy function along the slip line from the side of the subsonic flow. The function φ_1 is determined by the relation for φ from equation (10) using the slope angle σ of the external shock to the axis of symmetry at the point C; the latter is obtained from integration of equation (14) in the previous part of the impingement region.

The system of equations (16), (17), (6) and (8) is used to determine the four unknown values, namely M_0, M_1, θ and ε. For calculation of the coefficients of the polynomial in equation (8) we have two conditions at the point C:

$$\varepsilon=\varepsilon_C,\qquad \frac{d\varepsilon}{dr}=-\operatorname{ctg}\theta_C,\qquad r=r_C, \quad (18)$$

where ε_C and r_C are the coordinates of the point C. θ_C is the slope angle of the slip line to the z-axis at the point C.

The third condition is obtained at the slip line in the vicinity of the sonic point with the coordinates denoted by (r_{cr}, h). Upon integration of equation (16) for the derivative of the Mach number M_1 it may be found that this equation has a singular point at $\theta = \pi/2$. It may be easily shown that the latter point is a "saddle-type" point. From the regularity of the solution in the vicinity of the singular point, it follows that

$$-M^2\pi(M)\frac{d\theta}{dr} - \frac{2}{h\varkappa}\left[\pi(M) - \pi(M_0)\times\left(\frac{\varphi_0}{\varphi_1}\right)^{-1/(\varkappa-1)}\right] = 0. \quad (19)$$

$$\bar{q} = \frac{\varphi q(M_B)\displaystyle\int_0^{r_c}\frac{(L-\varepsilon)r\,dr}{\{\sqrt{[r^2+(C-\varepsilon)^2]}+(\sqrt{\varphi})\}^2\sqrt{[r^2+(L-\varepsilon)^2]}}}{\frac{1}{2}\left\{1+\sqrt{\left(\frac{2}{\varkappa-1}\right)}\left[\frac{\varkappa+1}{2}\left(\frac{\varphi_1}{\varphi_0}\right)^{1/\varkappa}(2h\varkappa a_1+2)^{-(\varkappa-1)/\varkappa}-1\right]^{\frac{1}{2}}\left(\frac{\varphi_1}{\varphi_0}\right)^{\frac{1}{2\varkappa}}(2h\varkappa a_1+2)^{1/\varkappa}\right\}}$$

The above equation is used to determine M_0 at the normal to the point T (Fig. 1). Taking approximately that the sonic point at the slip line coincides with the singular point we get from equation (19) with the approximation (8) the following expression

$$M_0^2 = \frac{\varkappa+1}{\varkappa-1}\left(\frac{\varphi_1}{\varphi_0}\right)^{1/\varkappa}(2h\varkappa a_1+2)^{-(\varkappa-1)/\varkappa} - \frac{2}{\varkappa-1}. \quad (20)$$

For determination of the coordinates of the point T, the law of mass continuity is to be used for the critical section $r = r_{cr}$ and the circular section II parallel to the plate through the point C (Fig. 1)

$$\frac{Q}{2\pi} = \int_0^{r_C} r\rho V\cos\vartheta\,dr = r_{cr}\int_0^h \rho_{cr}V_{cr}\,dz, \quad (21)$$

where ρ, V, ϑ are the flow parameters at the section II, ρ_{cr}, V_{cr} are the same at the critical section.

Using equation (12), calculation of the first integral in equation (21) is a straightforward procedure; as to the second integral in equation (21), we approximate its element of integration by the linear function of the type

$$\rho V = \rho_0 V_0 + \frac{\rho_1 V_1 - \rho_0 V_0}{h}. \quad (22)$$

Then expressing ρ and V in equation (22) in terms of M and using the condition $M_1 = 1$ at the critical section and M_0 from

$$r_{cr}h = \bar{q}, \quad (23)$$

where

Equality (23) together with the conditions at the sonic point makes the system of equations (16), (17), (6) and (8) closed. The initial conditions for integration of this system are those received from integration of the previous system of equations and from calculation of the triple point.

Singularity points present make a peculiarity of the latter system.

In addition to the saddle point $\theta = \pi/2$ there is a singular point $M_0 = 1{\cdot}0$. Formally the regularity of the solution of equation (16) in the vicinity of the point $\theta = \pi/2$ is secured by setting up the approximate expression for slip line (6), however, this regularity actually exists only when the regularity of the solution of equation (17) in the vicinity of the point $M_0 = 1{\cdot}0$ is secured. This may be done by the appropriate choice of the parameter ε_0.

A numerical solution of the above system of equations has been obtained by the Runge–Kutta method. The algorithm of the solution consists in the reiterative successive integration of the systems of equations with the purpose to determine the only unknown parameter ε_0.

It is worth mentioning that the method of integral relationships used here as well as any of the other known approximate methods such as approximation of the initial functions by the Lagrange polynomials used in [5] does not satisfy the compatibility condition at the triple point. This may probably be attributed to errors of the method or to a more complicated flow pattern in the vicinity of the triple point than that taken at the present stage. The numerical calculations show the best agreement between the theory and experiment on the central shock detachment (ε_0) when the triple point is calculated from the condition of the pressure equality in the flow along the two sides of the slip line and at the initial slope angle θ_C equal to that at the point C from the side of the subsonic region. In Fig. 2 the results of the calculations are

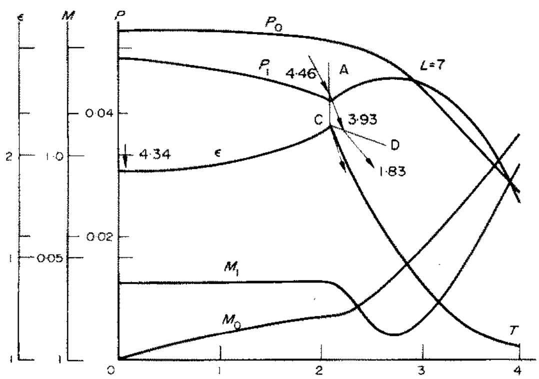

FIG. 2. The changes of the flow parameters along the plate and gas dynamic lines against the radial distance r for a jet with $M_a = 2{\cdot}07$; $n = 8{\cdot}2$; $\varkappa = 1{\cdot}25$; $L = 7$.

presented for the following jet parameters: $M_a = 2{\cdot}07$; $n = 8{\cdot}2$; $\varkappa = 1{\cdot}25$; and nozzle-to-plate spacing $L = 7$.

As follows from the solution, the nonuniformity (and hence the vorticity) of the flow considerably increases in the vicinity of the triple point. The existence of a peripheral maximum in the velocity component normal to the plate near the triple point induces the flow drag coming from the stagnation point as the sense of the vortex rotation in the vicinity of the triple point is opposite to that of the flow along the plate.

As found in [6, 7], a back flow towards the stagnation point is possible at a certain ratio of the peripheral velocity to that in the region behind the central shock.

The gradient of the Mach number at the stagnation point is calculated from the flow field in the impingement region. For jets with $p_0 = 70\ \mathrm{kg/cm^2}$ the values of $(\mathrm{d}M_0/\mathrm{d}r)_{r=0}$ vs the nozzle-to-plate spacing L are plotted in Fig. 3.

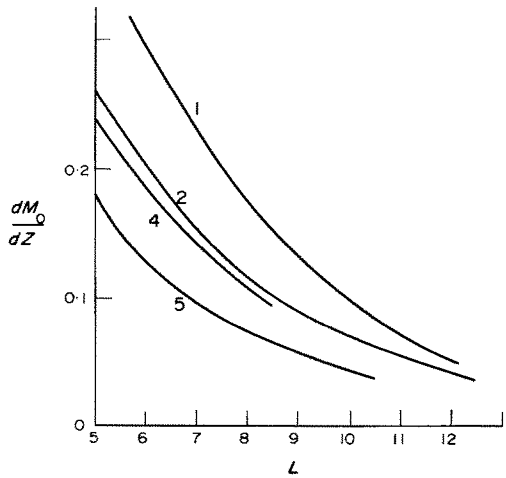

FIG. 3. The Mach number gradient at the stagnation point for different nozzle-to-plate spacings L. 1: $M_a = 1{\cdot}0$; $\varkappa = 1{\cdot}25$; $n = 39$; 2: 1·58; 1·25; 18·2; 3: 2·07; 1·25; 8·2; 4: 1·58; 1·4; 18·5.

With this value known, the dimensional velocity gradient in the vicinity of the stagnation point may be easily calculated

$$\beta = \sqrt{\left(\frac{\varkappa - 1}{2}\right)} \frac{\sqrt{(RT_0}}{r_a} \left(\frac{\mathrm{d}M_0}{\mathrm{d}r}\right)_{r=0}, \frac{1}{\sec}. \quad (24)$$

VISCOUS FLOW AND HEAT TRANSFER IN A VISCOUS MIXING REGION NEAR THE PLATE

The calculation of the viscous flow and heat transfer is restricted with the subsonic region POECT (Fig. 1). With the dissipative terms neglected, the governing equations are

$$\frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} + \frac{v_r}{r} = 0, \quad (25)$$

$$v_r \frac{\partial \Omega}{\partial r} + v_z \frac{\partial \Omega}{\partial z} - \frac{v_r \Omega}{r} = \Delta\Omega - \frac{\Omega}{r^2},$$

$$\Omega = \frac{\partial v_z}{\partial r} - \frac{\partial v_r}{\partial z}, \tag{26}$$

$$v_r \frac{\partial T}{\partial r} + v_z \frac{\partial T}{\partial z} = \frac{1}{Pr}\left[\frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right], \tag{27}$$

where using the ordinary transformations for the flow near the stagnation point v_r and v_z are based on $\sqrt{(\beta \nu)}$, Ω on β, the linear dimensions on $\sqrt{(\nu/\beta)}$, β is the dimensional velocity gradient at the stagnation point (see equation (24)), $T = [(\bar{T}_w - T)/(\bar{T}_w - \bar{T}_\infty)]$, $\bar{T}$ is the instantaneous temperature, $\bar{T}_w$ and $\bar{T}_\infty$, the wall temperature and that of an undisturbed flow (far from the wall), are taken to be constant, ν is the kinematic viscosity.

The vorticity of the flow along the gas dynamic line ECT is expressed as the Bessel series [8]

$$\Omega_1 = \sum_{n=1}^{\infty} A_n k_n z J_1(k_n r), \tag{28}$$

where J_1 is the first order Bessel function of the first kind.

Expression (28) may physically be interpreted as the expression of Ω_1 in the form of a spectrum of the vortices with the wave lengths $\lambda_1, \lambda_2, \ldots, \lambda_n$ distributed along r where λ_1 is chosen to be the largest or the main wave length of the spectrum. The total effect of the above spectrum is equal to the effect of the vortex Ω_1 in the external flow. In equation (28) $k_n = 2\alpha_n/\lambda_1$; α_n is the nth root of the equation $J_1(\alpha) = 0$. From the equation relating the streamfunction with the vortex

$$\Delta\psi = -\Omega_1 r$$

it follows that

$$\psi(r, z) = r^2 z + \sum_{n=1}^{\infty} \frac{A_n}{k_n} r z J_1(k_n r). \tag{29}$$

In equation (29) the first term is the streamfunction of the potential motion and the second one is the streamfunction of the disturbed motion due to vorticity (nonuniformity) of the external flow. The coefficients A_n of the series are to be calculated in every particular case of the jet impingement from the solution of the ideal problem, k_n may be considered as a nondimensional wave number, of the nth vortex. From the definition of the wave number

$$k_n \lambda_n = k_1 \lambda_1 = \text{const.}$$

With account for equation (29) the solution in the viscous mixing layer near the plate is to be sought in the form

$$\psi = r^2 f_0(z) + \sum_{n=1}^{\infty} \frac{1}{k_n} f_n(z) r J_1(k_n r) \tag{30}$$

(for the streamfunction),

$$T = \theta_0(z) + \sum_{n=1}^{\infty} \theta_n(z) J_0(k_n r) \tag{31}$$

(for the temperature).

The boundary conditions for f_0, f_n, θ_0, θ_n are

$$f_0(0) = f_0'(0) = 0, \quad f_0'(\infty) = 1, \quad \theta_0(0) = 0, \quad \theta_0(\infty) = 1,$$

$$f_n(0) = f_n'(0) = 0, \quad f_n'(\infty) = A_n,$$

$$\theta_n(0) = \theta_n(\infty) = 0, \quad (n = 1, 2, 3, \ldots \infty). \tag{32}$$

According to equation (30) the expressions for the velocity components and the vorticity are written as

$$v_z = -2f_0 - \sum_{n=1}^{\infty} f_n J_0(k_n r),$$

$$v_r = r f_0' + \sum_{n=1}^{\infty} \frac{1}{k_n} f_n' J_1(k_n r),$$

$$\Omega = r f_0'' + \sum_{n=1}^{\infty} \left(\frac{f_n''}{k_n} - k_n f_n\right) J_1(k_n r). \tag{33}$$

Consideration of the shape of the distribution of the velocity component normal to the plate along the line ECT for a jet with $n = 8{\cdot}2$;

$L = 7$; $M_a = 2{\cdot}07$ shows that for a qualitative analysis only one term $A_1 \simeq 0{\cdot}5$ in the spectrum (28) may be taken with the values A_n at $n = 2, 3, \ldots \infty$ neglected. The value of the wave number corresponding to the first harmonic of the spectrum k_1 has the order of magnitude of about 10^{-3}. Then substituting equations (33) and (31) into equations (25)–(27) and neglecting the terms with k_n^2 yield

$$r^2(f_0''' + 2f_0f_0'' - f_0'^2 + 1) + \sum_n [f_n''' + 2(f_0f_n'')]$$
$$\times \frac{r}{k_n}(k_nr) - \sum_n (f_0'f_n' - f_0''f_n - A_1)r^2J_0(k_nr)$$
$$- \sum_{n,i} \frac{1}{k_n}(f_n'f_i' - f_if_n'' - A_1^2)rJ_0(k_ir)J_1(k_nr)$$
$$+ \sum_{n,i} \frac{1}{k_ik_n}(f_i' - f_n' - A_1^2)J_1(k_ir)J_1(kn_r)]$$
$$\equiv N(r, z) = 0, \qquad (34)$$

$$\frac{1}{Pr}T_0'' + 2f_0T_0' + \sum_n \left(\frac{1}{Pr}\theta_n'' + 2f_0\theta_n' + T_0'f_n\right)$$
$$J_0(k_nr) + f_0'\sum_n \theta_nk_nrJ_1(k_nr) + \sum_{n,i} f_n\theta_i'J_0(k_1r)J_0(k_nr)$$
$$+ \sum_{n,i} \frac{k_i}{k_n} f_n'\theta_iJ_1(k_ir)\,J_1(k_nr) \equiv M(r,z). \quad (35)$$

Summation over n, i in equations (34) and (35) is made for such n, i for which the condition k_n, $k_i \ll 1$ is still fulfilled. For the problem solution we apply the finite Hankel transformation to the expressions $N(r, z)$ and $M(r, z)$ within the range $[0, \lambda_1/2]$. Substituting the variables $r = (\lambda_1/2)x$, $0 \leqslant x \leqslant 1$ with the notations $\alpha_n = \xi$, we get

$$\int_0^1 N(x, z)xJ_0(\xi, x)\,dx = 0,$$
$$\int_0^1 M(x, z)xJ_0(\xi, x)\,\mathrm{d}x = 0,$$
$$(\xi = 0; 3{\cdot}83; 7{\cdot}015; 10{\cdot}17\ldots) \qquad (36)$$

After calculation of the above integrals in equation (36) at different values of ξ $2(n + 1)$ equations are obtained for the functions f_0, T_0, f_n, θ_n. The accuracy of the calculations is shown to be high enough if two terms in the series (30) and (31) are only taken.

In Fig. 4 the results of the calculation of the heat flux towards the plate q_w based on the corresponding value for the uniform direct

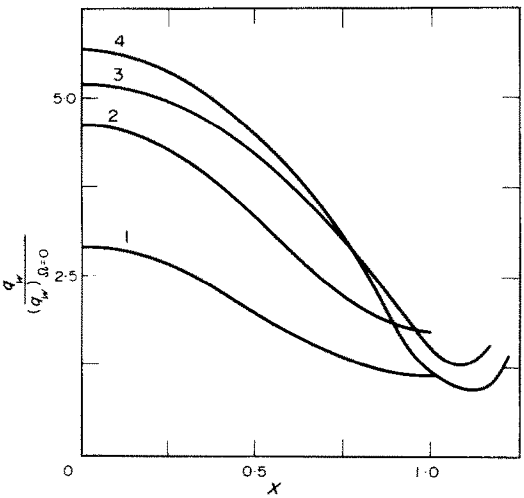

FIG. 4. The relative heat flux distribution against the radial direction $x = 2r/\lambda_1$. 1 (theory): $A_1 = 0{\cdot}5$; $K_1 = 10^{-3}$; 2: 1·0; 10^{-3}. 3 (experiment): $M_a = 2{\cdot}07$; $n = 8{\cdot}2$; $\varkappa = 1{\cdot}25$; $L = 7$; 4: 1·58; 18·2; 1·25; 7.

stagnation-point flow $(q_w)_{\Omega=0}$ are plotted vs the radial direction x. Here are presented also the experimental data on heat flux towards the plate for a jet with $n = 8{\cdot}2$; $M_a = 2{\cdot}07$; $L = 7$ with the vorticity parameters $A_1 = 0{\cdot}5$; $k_1 = 10^{-3}$.

The calculation has revealed that the flow nonuniformity considerably increases the heat transfer between the jet and the plate compared with the uniform flow. Any increase in the flow nonuniformity (see $A_1 = 1{\cdot}0$ in Fig. 4) results in an increase of the heat flux. The disagreement between the presented calculations and the experimental data may probably be attributed to the effect of the jet turbulence and the flow dependence on time.

REFERENCES

1. I. P. GINZBURG, I. A. BELOV, V. A. ZAZIMKO and V. S. TERPIGOREV, On turbulence characteristic effects on heat transfer between a supersonic jet and a flat plate, *Heat and Mass Transfer*, Vol. 1, pp. 381–394. Energiya, Moscow (1968).
2. G. I. AVERENKOVA and E. A. ASHRATOV, Supersonic jet outflow into vacuum, *Calculation Methods and Programming*, Vol. 7, pp. 225–241. MGU, Moscow (1967).
3. I. P. GINZBURG, *Aerodynamics*. Vyssh. Shkola, Moscow (1965).
4. G. S. ROSLYAKOV, Interaction of plane shocks of the same directions, *Numerical Methods in Gas Dynamics*, Vol. 4, pp. 28–51. MGU, Moscow (1965).
5. S. M. GILINSKY, G. F. TELENIN and G. P. TINYAKOV, Calculation methods of a supersonic flow around blunt bodies with a detached shock wave, *Izv. AN SSSR, Mekhan. i Mashinostr.*, No. 4, 9–28 (1964).
6. I. P. GINZBURG, B. G. SEMILETENKO, B. S. TERPIGOREV and V. N. USKOV, Some peculiarities of interaction between supersonic underexpanded jet with a flat plate. *Inzh.-Fiz. Zh.* **19**(3), 412–417 (1970).
7. O. I. GUBANOVA, V. V. LUNEV and L. I. PLASTININA, On central stalling region at interaction of underexpanded supersonic jet with a plate, *Izv. AN SSSR, Mekh. Zhidk. i Gaza*, No. 2, 135–138 (1971).
8. I. A. BELOV and L. I. SHUB, Vortex flow at the stagnation point, *Izv. AN SSSR, Mekh. Zhidk. i Gaza*, No. 6, 85–89 (1970).

INTERACTION D'UN JET SUPERSONIQUE SOUSEXTENSIF AVEC UN OBSTACLE

Résumé—La solution numérique du problème d'une interaction du jet symétrique supersonique sousextensif avec un obstacle plan installé perpendiculairement à l'axe de jet dans les limites de sa partie initiale a été presentée. Les résultats de la calculation obtenus en une formulation ainsi posée du fluide idéal montrent qu'il existe un tourbillon considérable du fluide dans une région subsonique d'un écoulement près de l'obstacle. Les quantités obtenues d'un gradient de la vitesse radiale à proximité du point d'enraiement et la distribution du tourbillon sont employés ensuite pour la calculation d'un écoulement du fluide visqueux et le flux de chaleur au moyen de l'obstacle. On trouve que le flux de chaleur à l'obstacle est de 3 à 5 fois plus que celui expliqué par la théorie d'un écoulement unidimensionnel à proximité du point d'enraiement; le dernier fait est en accord avec les données expérimentales disponibles.

ZUSAMMENWIRKEN DES UNVERBREITERTEN ÜBERSCHALLSTRAHLS MIT EINEM HINDERNIS

Zusammenfassung—Im Arbeit wird die Lösung der Aufgabe über die Strömung der Flüssigkeit im Gebiet der Minimaleinwirkung beim Zusammenwirken des Überschallstrahls mit einem flachen senkrechten Hindernis innerhalb der Grenzen des Anfangsabschichts angeführt. Die in der Voraussetzung der reibungslosen Flüssigkeit durchgeführte Rechnungen zeigten die Existenz wesentlichen Wirbelstroms im Unterschallströmungsgebiet. Die ergebenen Werte des Geschwindigkeitsgradients im Bremsepunkt und Grössen des Wirbels wurden dann für die Rechnung der klebrigen Strömung und des Wärmedurchgangs im Bremsepunktgebiet ausgenutzt. Als Resultat der Lösung wird festgestellt, dass die Grösse des Wärmeflusses im Bremsepunkt den entsprechenden Wert für den wirbelfreien Strom um 3–5 Male übertraf, wenn die Wirbelbewegung des einfallenden Stromes in Betracht gezogen wird. Diese Tatsache wird durch Versuchsangaben bestätigt.

ВЗАИМОДЕЙСТВИЕ НЕДОРАСШИРЕННОЙ СВЕРЗВУКОВОЙ СТРУИ С ПРЕГРАЦОЙ

Аннотация—Приводится приближенное решение задачи о течении жидкости в области минимального влияния при взаимодействии сверхзвуковой недорасширенной струи с плоской нормально расположенной преградой в пределах начального участка струи. Результаты расчета в предположении идеальности жидкости показали существование значительной завихренности в области дозвукового течения. Полученные значения градиента скорости в точке торможения и величины вихря затем использованы для расчета вязкого течения и теплообмена в области вблизи преграды. В результате решения установлено, что величина теплового потока в точке торможения, определенная с учетом завихренности потока струи, превосходит соответствующее значение для равномерного потока в 3–5 раз; последний факт подтверждается экспериментальными данными.